

VEP oscillation solutions to the solar neutrino problem

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Abstract

We study the solar neutrino problem within the framework of a parametrized post-Newtonian formulation for the gravitational interaction of the neutrinos, which incorporates a violation to the equivalence principle (VEP). Using the current data on the rates and the energy spectrum we find two possible oscillation solutions, both for a large mixing angle. One of them involves the MSW effect in matter and the other corresponds to vacuum oscillations. An interesting characteristic of this mechanism is that it predicts a semi-annual variation of the neutrino flux. Our analysis provides new constraints for some VEP parameters.

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I. INTRODUCTION

Several experiments sensitive to solar neutrinos have measured a ν_e flux, with results lower than the values predicted by standard solar models (SSM) for different neutrino energies: the Homestake Cl radiochemical experiment [1], with sensitivity down to the lower part of the ^8B spectrum and the ^7Be line, the two radiochemical ^{71}Ga experiments, GALLEX [2] and SAGE [3], which are sensitive to the low energy pp neutrinos and above, and the water Cerenkov experiments, Kamiokande [4] and SuperKamiokande (SK) [5], which can observe only the highest energy ^8B neutrinos. A combination of any two of the experiments disfavors an astrophysical solution to the problem, and seems to indicate that a non-standard physical process is modifying the energy spectrum of solar neutrinos.

A widely accepted explanation of the discrepancy is based on the assumption that non-degenerate massive neutrinos do undergo flavor oscillations, either in vacuum or within the Sun (MSW effect) [6]. Another less orthodox mechanism for neutrino oscillations, which does not need neutrinos to have a mass, was proposed several years ago [7] and requires the coupling of neutrinos to gravity to be flavor dependent, i.e., a violation of the equivalence principle (VEP) in the neutrino sector. Some phenomenological consequences of this mechanism have been examined in a number of papers [8–14].

In a recent work [15] we developed a generalized VEP mechanism for neutrino oscillations, which is based on an extended parametrized post-Newtonian formalism (PPN). Here we apply this approach to the concrete situation of solar neutrinos, and in particular to the analysis of the seasonal variation of the signal. Using the latest data on total rates from the five experiments, and those on the energy spectrum and the seasonal variations from SK, we determine the allowed regions and the best-fits values for the oscillation parameters. We show that a solution to the solar neutrino problem is possible within the VEP scheme, not only for MSW matter-enhanced transformations but also for vacuum oscillations.

In the solar system the gravitational field receives contributions from several sources. Assuming, as is commonly done, that the potential vanishes at an infinite distance from the

source, the dominant contribution is given by the Great Attractor, with small perturbations produced by galactic clusters, our galaxy, and the Sun. Consequently, it is reasonable to approximate the potential by a constant of the order of 10^{-5} [16]. The effect of this potential regarding a possible VEP mechanism has already been analyzed by Halprin, Leung, and Pantaleone [13]. Here we follow the more general approach of Ref. [15], which incorporates not only a possible flavor dependence of the gravitational couplings, but also the most general violations to Einstein gravity in the context of metric theories. In this approach, the metric is given by the Minkowskian one plus source dependent perturbations: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

The assumptions for constructing the metric in the PPN formalism involve virialized sources such that $\frac{M}{R} \sim w^2$, where the quantities M, R , and w represent estimations of the order of magnitude of the mass, distance and characteristic velocity of the source. In what follows we keep only first order corrections to the flat space-time metric $\eta_{\mu\nu}$, and we neglect a possible angular momentum of the Great Attractor, which in any case would lead to very small corrections. Thus we have $h_{oi} = 0$, while the non-null corrections are given by

$$h_{oo} = 2\gamma'U + \mathcal{O}(w^4) , \quad (1)$$

$$h_{ij} = 2\gamma U\delta_{ij} + \Gamma U_{ij} + \mathcal{O}(w^4) , \quad (2)$$

where γ, γ' , and Γ are adimensional parameters of the PPN expansion (up to order w^3). In the particular case of Einstein gravity we have $\Gamma = 0$, and $\gamma = \gamma' = 1$. The potentials responsible for the metric perturbations are

$$U = \int \frac{\rho(\mathbf{r}') d^3r'}{|\mathbf{r} - \mathbf{r}'|} , \quad U_{ij} = \int \frac{\rho(\mathbf{r}')(r_i - r'_i)(r_j - r'_j) d^3r'}{|\mathbf{r} - \mathbf{r}'|^3} , \quad (3)$$

with $\rho(\mathbf{r})$ being the mass density of the source of the gravitational field. We are using a system of unities with $G = \hbar = c = 1$.

For a confined and distant source, U can be approximated by

$$U \approx \frac{M}{R} + O\left(\frac{1}{R^2}\right) . \quad (4)$$

If we take the z-axis along the direction determined by the solar system and the gravitational source, we then have $U_{zz} \sim U$. However, the components U_{xz} and U_{yz} are proportional to

$\Delta\theta U$, where $\Delta\theta$ is the angular size of the source, while U_{xx} , U_{yy} , and U_{xy} are of the order of $(\Delta\theta)^2 U$. Since the Great Attractor is a rather extended object with an angular size of the order of 10^{-1} [17], in the case of the solar system there are only three relevant types of U_{ij} contributions: (i) those coming from our galaxy, which are of order 10^{-6} , (ii) a longitudinal component from the Great Attractor, of order $U_{zz} \simeq U \simeq 10^{-5}$, and (iii) transverse-longitudinal components also produced by the Great Attractor, of the same order as the galactic contributions, $U_{xz} \simeq U_{yz} \simeq 10^{-6}$. Therefore, possible VEP flavor oscillations of solar neutrinos would be characterized by three main effects: an isotropic effect ($U \simeq 10^{-5}$), and two anisotropic effects ($U_{zz} \simeq 10^{-5}$, $U_{ij} \simeq 10^{-6}$). In the next section we review the essential ingredients of the VEP mechanism for neutrino oscillations within the context of the PPN formalism, and in Section III we apply it to the study of the solar neutrino problem.

II. VEP INDUCED OSCILLATIONS

For simplicity, in what follows we consider that there are only two (massless) neutrino flavors, ν_e and ν_μ . In our VEP scenario they are assumed to be linear superpositions of the gravitational eigenstates ν_1^g and ν_2^g , with a mixing angle θ_g . Each gravitational eigenstate is characterized by a different set of PPN parameters, $\{\gamma^a, \gamma'^a, \Gamma^a\}$ ($a = 1, 2$). This leads to different dispersion relations for the ν_a^g , which can be approximated by [15]

$$E^a = p \left[1 - (\gamma'^a + \gamma^a)U - \Gamma^a U_{ij} \frac{p_i p_j}{p^2} \right]. \quad (5)$$

Suppose that the initial state produced at time t_0 corresponds to a pure electron neutrino. Then, for a constant gravitational field the survival probability after traveling a distance $L = t - t_0$ is

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_g \sin^2 \frac{\pi L}{\lambda_g}. \quad (6)$$

According to this, neutrino oscillations will appear whenever a non null mixing is generated

because of flavor dependent gravitational interactions. These oscillations have a characteristic length given by

$$\lambda_g = \frac{2\pi}{|\Delta_0|} , \quad (7)$$

with

$$\Delta_0 = E_2 - E_1 = -E \left[(\delta\gamma' + \delta\gamma)U + \delta\Gamma U_{ij} \frac{p_i p_j}{E^2} \right] , \quad (8)$$

where $E \simeq p$ is the neutrino beam energy, and

$$\delta\gamma = \gamma^2 - \gamma^1, \quad \delta\gamma' = \gamma'^2 - \gamma'^1, \quad \delta\Gamma = \Gamma^2 - \Gamma^1. \quad (9)$$

In contrast to the ordinary vacuum oscillations induced by a mass difference, where $\lambda_m = 4\pi E/\delta m^2$ is proportional to the energy, the effect we are considering here has an oscillation length that goes with E^{-1} . This leads to observable distinctions between both mechanisms and makes the gravitational induced oscillations suitable to be observed with higher energy neutrinos [9,12]. Note that even though the overall sign of the gravitational potential is irrelevant for oscillations, the relative signs among differences of the PPN parameters are very significant. If we assume that these differences are all of the same order, then the most important directional effect would be given by the quadrupolar contribution corresponding to U_{zz} .

As is well known, flavor transformations of massive neutrinos are affected by their interactions with matter [18]. Neutral current interactions are flavor diagonal and can be ignored, as long as we do not consider sterile neutrinos and neutrinos are not part of the medium [19], but this is not true for the charged current interactions. As a consequence, the forward scattering amplitude is not flavor diagonal and depends on the leptonic content of the matter, which gives place to important consequences such as the MSW effect. A similar phenomenon happens for the VEP mechanism in the presence of matter. In this case, the flavor evolution for relativistic neutrinos propagating through a constant gravitational field is governed by the equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \mathcal{H}(t) \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad (10)$$

where the Hamiltonian $\mathcal{H}(t)$, after discarding an irrelevant overall phase, can be written as

$$\mathcal{H}(t) = \frac{\Delta_0}{2} \begin{pmatrix} -\cos 2\theta_g & \sin 2\theta_g \\ \sin 2\theta_g & \cos 2\theta_g \end{pmatrix} + \frac{b(t)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (11)$$

The first term arises from VEP and the second term accounts for the matter effects on the neutrino propagation. For a normal matter background, as in the case of the Sun, we have $b(t) = \sqrt{2}G_F N_e(t)$, where G_F is the Fermi constant and $N_e(t)$ denotes the electron number density. An extended gravity like the one here considered could also affect the electroweak Lagrangian, but the combined effect should be of the order UG_F . Therefore, they are strongly suppressed and we do not include them in our discussion.

Taking into account the dominant contributions due to U and $U_{zz} \simeq U$, the coefficient Δ_0 reduces to

$$\Delta_0 = -EU \left[(\delta\gamma + \delta\gamma') + \delta\Gamma \cos^2 D \cos^2 (\alpha - A) \right]. \quad (12)$$

Here, α is the right ascension of the Sun, and A and D are the right ascension and declination of the Great Attractor in ecliptic coordinates. The second term in Δ_0 arises from the quadrupolar potential of the gravitational source and generates a seasonal dependence in the oscillation wavelength, as first discussed in Ref. [15]. This effect went unnoticed in previous work on the subject [9–13], where only the contribution coming from the Newtonian gravitational potential was considered. To isolate the anisotropic contribution, it is convenient to reparametrize Δ_0 as follows

$$\Delta_0 = -EU\delta\bar{\gamma} \left[1 + \left(\cos^2 (\alpha - A) - \frac{1}{2} \right) \delta \right] \quad (13)$$

where $\delta\bar{\gamma} = (\delta\gamma + \delta\gamma')/(1 - \delta/2)$ and $\delta = \delta\Gamma \cos^2 D / \delta\bar{\gamma}$, so that the annual average of Δ_0 is independent of δ . We will define δ positive, because (δ, A) is equivalent to $(-\delta, A + \pi/2)$.

At any time, $\mathcal{H}(t)$ can be diagonalized by a unitary transformation characterized by an angle $\theta_m(t)$

$$\sin 2\theta_m(t) = \frac{\Delta_0 \sin 2\theta_g}{\sqrt{(\Delta_0 \cos 2\theta_g - b_e(t))^2 + (\Delta_0 \sin 2\theta_g)^2}}. \quad (14)$$

There exists a resonant flavor conversion when the diagonal elements of the Hamiltonian vanish, i.e., when

$$\sqrt{2}G_F N_e(t_R) = \Delta_0 \cos 2\theta_g, \quad (15)$$

and in this case the mixing in matter is maximal ($\sin 2\theta_m = 1$).

The efficiency of the conversion mechanism depends on the adiabaticity of the process. For a constant gravitational field, the average probability for a ν_e produced in the Sun to reach the Earth reads

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \frac{1}{2}(1 - 2P_c) \cos 2\theta_m^0 \cos 2\theta_g, \quad (16)$$

with $\theta_m^0 = \theta_m(t_0)$. The function P_c represents the probability of transition between the instantaneous eigenstates of $\mathcal{H}(t)$. It embodies the total correction to the adiabatic result for $\langle P_{\nu_e} \rangle$, which corresponds to $P_c = 0$.

Except for regions close to the center and the surface, the electron density in the Sun is well approximated by an exponential profile [20]. Thus the change of the electron density along the path of a neutrino moving radially within the Sun can be written as

$$N_e(t) = N_e(t_0) \exp[-(t - t_0)/r_0], \quad t \geq t_0 \quad (17)$$

where $N_e(t_0)$ is the density at the production point and r_0 is a parameter to be adjusted according to the region [21]. In the SSM N_e takes its maximal value at the center of the Sun, where it is approximately equal to $100 N_A \text{ g/cm}^3$, where N_A is the Avogadro number. For $N_e(t)$ as given in Eq. (17), the following formula for P_c has been derived in a given approximation from the exact analytical solution of the evolution equation [22]

$$P_c = \frac{\exp \left[\pi \kappa \left(\frac{\cos 2\theta_g}{1 - \cos 2\theta_g} \right) \right] - 1}{\exp \left[\pi \kappa \left(\frac{2 \cos 2\theta_g}{\sin^2 2\theta_g} \right) \right] - 1}, \quad (18)$$

where the adiabatic parameter κ is

$$\kappa = |\Delta_0| \frac{\sin 2\theta_g \tan 2\theta_g}{\frac{1}{N_e(t_R)} \left| \frac{dN_e(t)}{dt} \right|_{t_R}}. \quad (19)$$

In the denominator of the last formula we have discarded any term associated with variations of the gravitational field with the distance. From the above expressions, we see that $\bar{P}(\nu_e \rightarrow \nu_e)$ depends on the electron density in the production zone and the logarithmic derivative of the density in the transition layer.

For $\kappa \gg 1$, P_c is exponentially small. On the other hand, when $\kappa < 1$ there are considerable corrections to the adiabatic approximation that reduces the magnitude of the resonant transformation. Nonadiabatic effects become important when κ is of order 1, provided that the neutrinos go through a resonance. If $b(t_0) < \Delta_0 \cos 2\theta$, level crossing cannot occur, $P_c = 0$ and neutrino propagation will be adiabatic even for $\kappa < 1$. An effective way to account for this situation is to multiply the expression of Eq. (18) by the step function $\Theta(b(t_0) - \Delta_0 \cos 2\theta)$, so that the transition probability vanishes when neutrinos are produced below the resonance. The MSW survival probability $\langle P_{\nu_e} \rangle$, averaged on the production region for the ^8B neutrinos, is plotted in Fig. 1 as a function of Δ_0 for different mixing angles. These curves have been obtained using the electron density predicted by the SSM [20,23]. Notice that, in contrast with the common MSW mechanism for massive neutrinos, in the case of VEP the adiabatic edge is at higher energies, whereas the nonadiabatic edge is at lower energies [12]. The adiabatic edge shifts towards higher energies with decreasing mixing angle, as seen from Eq. (19) and the condition $\kappa \gg 1$.

III. NEUTRINO EVENT RATE AND ENERGY SPECTRUM

In the presence of neutrino oscillations, the capture rate for the radiochemical experiments, such as ^{37}Cl and ^{71}Ga , is given by:

$$R(E) = g(t) \sum_k \int_0^\infty dE_\nu \Phi_k(E_\nu) \langle P_{\nu_e} \rangle \sigma(E_\nu) dE_\nu, \quad (20)$$

where $\sigma(E_\nu)$ is the cross section for neutrino capture and $\Phi_k(E_\nu)$ is the k -component of neutrino flux spectrum. Here, $g(t)$ is a geometrical factor due to the Earth's orbit eccentricity

and $\langle P_{\nu_e} \rangle$ is the survival probability averaged over the production regions for the different neutrino sources.

For neutrino-electron scattering experiments, such as SK, the solar neutrino induced event rate can be written:

$$R(E) = g(t) \int_{-\infty}^{\infty} dE_e \Xi(E_e, E) \int_{E_{\nu \min}}^{\infty} dE_\nu \Phi(E_\nu) \times \left[\langle P_{\nu_e} \rangle \frac{d\sigma_e(E_\nu, E_e)}{dE_e} + (1 - \langle P_{\nu_e} \rangle) \frac{d\sigma_\mu(E_\nu, E_e)}{dE_e} \right], \quad (21)$$

where E_ν is the energy of the incident neutrino, E_e is the electron kinetic energy, and $\Phi(E_\nu)$ gives the neutrino flux spectrum. The function $\Xi(E_e, E)$ characterizes the Superkamiokande efficiency to measure the energy of the scattered electrons [5], and $d\sigma_\ell/dE_e$ ($\ell = e, \mu$) is the differential cross section for the $\nu_\ell - e$ elastic scattering, where E_e is the electron kinetic energy. This differential cross section can be calculated from the electroweak theory, and is given by

$$\frac{d\sigma_\ell}{dE_e} = \frac{\sigma_0}{m_e} \left[g_L^2 + g_R^2 \left(1 - \frac{E_e}{E_\nu} \right)^2 - g_L g_R \left(\frac{m_e E_e}{E_\nu^2} \right) \right], \quad (22)$$

with $\sigma_0 = 8.8 \times 10^{-45} \text{cm}^2$, $g_R = \sin^2 \theta_W$, and $g_L = \pm \frac{1}{2} + \sin^2 \theta_W$. The upper sign corresponds to $\nu_e - e$ and the lower sign to $\nu_\mu - e$ scattering, respectively. For the energy interval of solar neutrinos $d\sigma_\mu/dE_e \cong (0.155 - 0.166) d\sigma_e/dE_e$.

The VEP mechanism begins to be significant when half of an oscillation is about equal to the Sun-Earth distance. According to Eqs. (7) and (13), for a 10 MeV neutrino this corresponds to $|U\delta\bar{\gamma}| \approx 10^{-25}$, in which case we have pure vacuum oscillations. For larger values of $|U\delta\bar{\gamma}|$ the oscillation wavelength shortens, and when it becomes smaller than the solar radius the effect of the background matter turns out to be relevant through the MSW effect, with the mixing angle θ_m given by Eq.(14). To compute the event rate we follow in general the scheme of Ref. [6]. The ingredients used in our computation have been developed in different places. The matter effects on the calculation of $\langle P_{\nu_e} \rangle$ were incorporated by applying the analytic formula given by Eqs. (16) and (18), as discussed in Ref. [21]. The

electron density is given in Refs. [20] and [23], while the cross sections and the neutrino fluxes were obtained from Refs. [20] and [24].

We identify three regions in the $|U\delta\bar{\gamma}|$ - $\sin^2 2\theta$ parameter space for the VEP induced oscillations which are compatible with the observed total rates. Two of them correspond to MSW-enhanced VEP oscillations, whereas the third one is associated to vacuum VEP oscillations. The MSW solutions and the vacuum oscillation solution are separated by three orders of magnitude in $|U\delta\bar{\gamma}|$. To identify these regions we use a standard χ^2 analysis [25] of the data from all the solar neutrino experiments, taking into account both the experimental and theoretical errors.

As Fig. 2 shows, for the MSW VEP oscillations there are two 99% c.l. regions allowed by the measured rates in all the experiments. One of them is a small mixing angle solution, with $3.2 \times 10^{-3} \lesssim \sin^2(2\theta_g) \lesssim 5.7 \times 10^{-3}$ and $|U\delta\bar{\gamma}| \simeq 3.2 \times 10^{-19}$, and the other is a large mixing angle solution, with $0.6 \lesssim \sin^2(2\theta_g) \lesssim 1$ and $10^{-22} \lesssim |U\delta\bar{\gamma}| \lesssim 4 \times 10^{-21}$. The best fit for the small mixing angle is obtained with $\sin^2(2\theta_g) = 4 \times 10^{-3}$, whereas in the case of the large mixing angle it occurs at $|U\delta\bar{\gamma}| = 1.58 \times 10^{-22}$ and $\sin^2(2\theta_g) = 0.87$. At 94 c.l. the small mixing angle region disappears, and only the large mixing angle region remains.

Our analysis reveals that there is another allowed region, which corresponds to vacuum VEP oscillations and is shown in Fig. 3. At 99% c.l. the main sector is bounded by $0.75 \lesssim \sin^2(2\theta_g) \lesssim 1$ and $10^{-24} \lesssim |U\delta\bar{\gamma}| \lesssim 10^{-22}$. The values of the parameters for the best-fit point are $|U\delta\bar{\gamma}| = 1.82 \times 10^{-24}$ and $\sin^2(2\theta_g) = 1$. The MSW VEP solutions are consistent with those already found using the Newtonian approximation for the gravitational interaction [12,13], while the new solution given by the vacuum VEP oscillations has been independently derived in a recent work [26]. In previous studies it has been argued that when half of an oscillation corresponds to the Sun-Earth distance for 10 MeV neutrinos the ^8B neutrinos are depleted but the lower-energy ^7Be neutrinos are unaffected, which is in contradiction with the experimental data [9,12]. However, a good agreement can be obtained if the wavelength is tuned for an energy E_t close to the energy of the Be line. In this way we have the required suppression of the lower-energy neutrinos, and due to the inverse energy

dependence of the oscillation length λ_g for VEP oscillations, we can also have a reduction by about 50% of the neutrino flux for higher energies, $E_d = nE_t$ with n integer. This is the origin of our vacuum VEP solution, with λ_g tuned for 1.13 MeV neutrinos.

Besides the total rate, the SK collaboration has provided spectral information on the ^8B solar neutrinos [27]. The measured energy spectrum of the scattered electrons is divided into bins having a width of 0.5 MeV in the range from 5.5 MeV to 14 MeV. An additional bin comprises of events with energy from 14 MeV to 20 MeV. Figs. 4 and 5 show the χ^2 analysis for the energy spectrum [6] corresponding to the MSW and vacuum VEP oscillations, and in Fig. 6 we display the spectrum of the best VEP solutions together with the experimental data. The small-angle MSW solution is excluded by the energy spectrum at 99% c.l., while both the vacuum solution and the large-angle MSW solution are allowed at 90% c.l. Figs. 7 and 8 display the χ^2 analysis carried out with the whole set of data, including simultaneously the total rate and the SK spectrum measurements, with the individual χ^2 treated as independent [6].

The eccentricity of the Earth's orbit produces a geometrical 7% variation of the neutrino flux since the Earth-Sun distance changes throughout the year. Due to the dependence of $\langle P_{\nu_e} \rangle$ on distance, an anomalous additional effect can be caused by the presence of the usual vacuum oscillations between massive neutrinos. Both effects are characterized by a one year period. Some indications of a seasonal variation in the neutrino flux from the Sun has already been seen in the GALLEX and Homestake experiments [28]. In Ref. [27], SK has also presented preliminary results that slightly favor a seasonal variation of the solar neutrino flux for $E_e > 11.5$ MeV in addition to the geometric variation. Within the present VEP oscillation scheme a non-geometrical seasonal variation of the flux is caused by the presence of the term proportional to $\delta\Gamma$ in Δ_0 (see Eq. (12)), which would produce a six month period variation. As a consequence, in contrast with the usual mass mechanism, the effect should be observed even in the case of MSW transformations. The authors of Ref. [26] conclude that no strong seasonal variation in the solar neutrino signal is expected for vacuum VEP oscillations. The difference with our result is due to the fact that in their analysis they follow

the common prescription to incorporate gravitational effects only through the Newtonian potential.

The seasonal variations of the flux above 11.5 MeV predicted by the best-fit VEP solutions including the anisotropic term are shown in Fig. 9, together with the SK data. All the solutions have a similar behavior and are compatible with the data within the present statistical accuracy. This is a consequence of the time resolution of the present experimental results. In principle, these solutions could be easily discriminated if the time resolution was improved, because their actual temporal dependence is very different, as Fig. 10 shows. The χ^2 analysis based on the data of SK for energies above 6.5 MeV and 11.5 MeV are shown in Fig. 11, in terms of the parameters δA and δ , where δA denotes the difference between the perihelion right ascension of the Sun and A modulo π . The best-fit solutions are: (a) $\delta A \simeq 140^\circ$ for MSW oscillations with large mixing angle, (b) $\delta A \simeq 60^\circ$ for MSW oscillations with small mixing angle, and (c) $\delta A \simeq 50^\circ$ for vacuum oscillations. Since 30° and 150° are the values of δA that are consistent with the position of the Great Attractor, the previous results seem to favour the large-angle MSW solution. This is not very conclusive because of the poor angular resolution of the data and the uncertainty concerning the position of the Great Attractor. Nevertheless, the analysis does give an improved boundary for the possible values of the parameter δ . At a 90% c.l., we have $\delta < 0.09$ for the small angle solution, and $\delta < 2$ for the large angle solution.

IV. FINAL REMARKS

We have used the present experimental result to re-examine the possibility that the VEP mechanism can provide a consistent solution to the solar neutrino problem. The total neutrino rates give three allowed regions: two of them correspond to a MSW solution, and the third one to a vacuum solution. The small-angle MSW solution is excluded at 94% c.l. and the large-angle MSW solution is discarded at 88% c.l. The most favored solution is given by the long-wavelength vacuum oscillations, whose best fit has a 35% c.l. The three

VEP solutions predict a seasonal dependence of the neutrino flux, in good agreement with the data. An improved time resolution is necessary in order to discriminate between the different periodic behaviors. The most restrictive conditions arise from the energy spectrum. In this case, the small-angle MSW solution is clearly ruled out by the SK data whereas the vacuum VEP oscillations are favored. The MSW oscillations with large mixing angle also remain as a possible solution.

More accurate data are necessary to properly establish the viability of the VEP mechanism as an adequate explanation of the solar neutrino deficit. However, the present analysis is sufficient to set new boundaries on the PPN coefficients that parametrize the violation of the equivalence principle. An improved resolution in the spectral measurements is required to establish the existence of a six-month period variation in the 8B neutrino flux, which is a signature of the VEP mechanism and makes a clear difference with other possible solutions, such as the standard vacuum oscillations of massive neutrinos [29].

Taking into account the different energy dependence of the VEP and mass mechanisms, a combination of both oscillations would give an excellent agreement with the experimental data. This possibility will be explored in a forthcoming paper.

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FIGURE CAPTIONS

Fig. 1: Neutrino survival probability for MSW VEP oscillations as a function of Δ_0 for different mixing angles. The probability has been averaged over the production region of the 8B neutrino.

Fig. 2: MSW solutions, total rates only. Allowed regions at 99% c.l. and 90% c.l. (darker shaded region) in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space. The best-fit point is indicated by a cross.

Fig. 3: Vacuum oscillations, total rates only. Allowed regions at 99% c.l. and 90% c.l. (darker shaded region) in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space. The best-fit point is indicated by a cross.

Fig. 4: MSW solutions, SK spectrum only. Excluded regions at 99% c.l. (darker shaded region) and 90% c.l. in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space. The best-fit point is indicated by a cross.

Fig. 5: Vacuum oscillations, SK spectrum only. Excluded regions at 99% c.l. (darker shaded region) and 90% c.l. in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space. The best-fit point is indicated by a cross.

Fig. 6: SK measured energy spectrum and best fits for the small mixing angle (SMA) MSW solution, large mixing angle (LMA) MSW solution, and vacuum oscillation (VO) solution. The solid line corresponds to the VO solution for the best fit of the spectrum data and the long-dashed line is the VO solution for the combined best fit of both the total rates and the spectrum data.

Fig. 7: MSW solution, total rates and energy spectrum. Allowed regions at 99% c.l. and 90% c.l. (darker shaded region) in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space.

Fig. 8: Vacuum oscillations, total rates and energy spectrum. Allowed regions at 99% c.l. and 90% c.l. (darker shaded region) in the $U\delta\bar{\gamma}\text{-sin}^2(2\theta_g)$ parameter space. The best-fit point is indicated by a cross.

Fig. 9: Seasonal variation of the flux above 11.5 MeV. We have plotted eight annual

bins for the different best-fit VEP solutions and for the SK data (black squares). The anisotropic effects due to $\delta\Gamma$ have been considered in the calculation. The solid line shows the geometrical variation due to the eccentricity of the Earth's orbit.

Fig. 10: Actual temporal variations of the flux for the different best-fit VEP solutions compared with the SK data.

Fig.11: Allowed 90% c.l and 35% c.l regions in the $(\delta-\delta A)$ plane for anisotropic VEP solutions: (a) VO solution, (b) SMA MSW solution, and (c) LMA MSW solution. The best-fit points are indicated by crosses.





















